Disequazioni Logaritmiche Esercizi

(P1)

Chiameremo $S_{C.E.}$ la soluzione del C.E. della disequazione logaritmica, dove bisogna porre gli argomenti dei logaritmi strettamente maggiori di 0 (>0)

$$\log_a A(x) + \log_a B(x) + \log_b C(x) > 3$$

$$S_{C.E.} = \begin{cases} A(x) > 0 \\ B(x) > 0 \\ C(x) > 0 \end{cases}$$

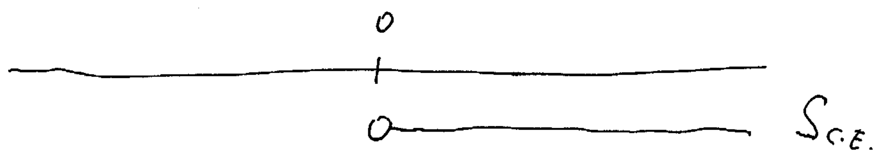
Chiameremo $S_{\log}$ la soluzione della disequazione logaritmica.

La Soluzione Totale $S_t = S_{C.E.} \cap S_{\log}$

Esercizio N 1

$$\log_{\frac{1}{2}} x < 4$$

C.E. : $x > 0$ $\qquad S_{C.E.} = x > 0$



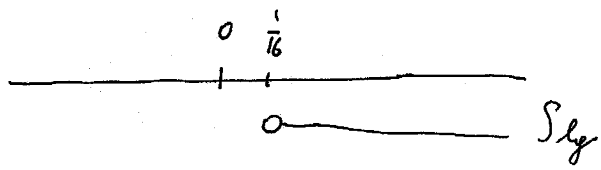
Ricerchiamo S_{lg}

Applichiamo la proprietà $\log_{\frac{1}{a}} b = -\log_a b$

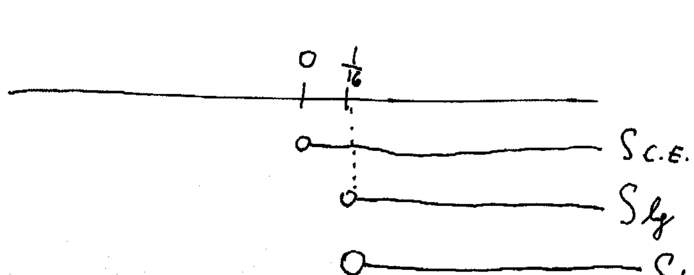
$-\log_2 x < 4$; $\log_2 x > -4$ applichiamo la funzione esponenziale di base 2.

$$\cancel{2^{\log_2 x}} > 2^{-4} \; ; \; x > 2^{-4} \; ; \; x > \frac{1}{2^4} \; ; \; x > \frac{1}{16}$$

$S_{lg}: x > \frac{1}{16}$



$S_t = S_{C.E.} \cap S_{lg} =$



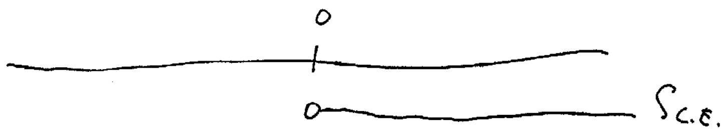
$S_t: \left]\frac{1}{16}; +\infty\right[$

Esercizio N2

p3

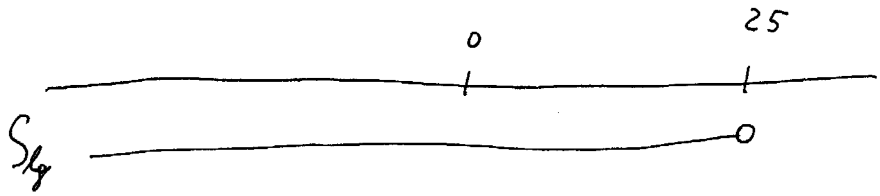
$$\log_{\sqrt[3]{5}} x < 6$$

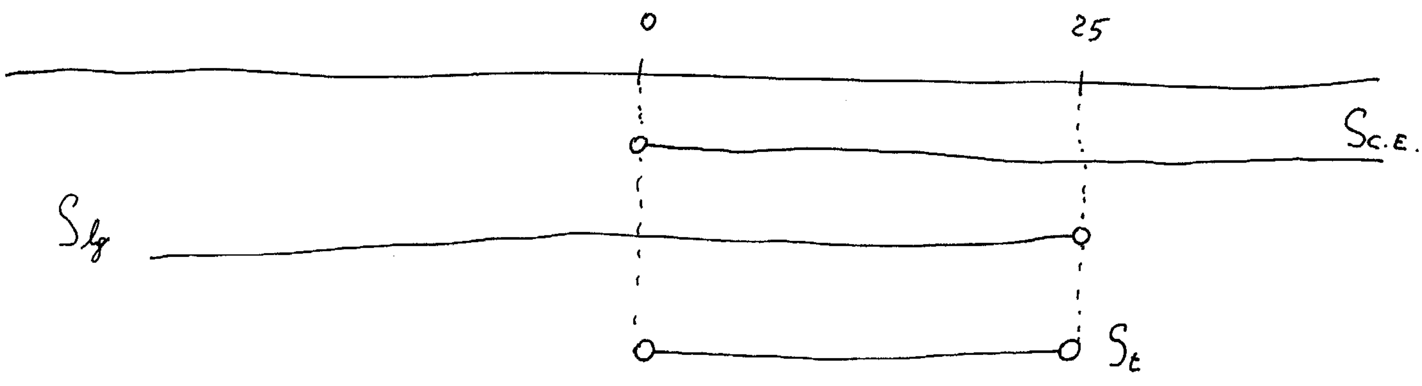
$\log_{5^{\frac{1}{3}}} x < 6$. $\qquad S_{c.e.} = x > 0$



$$5^{\frac{1}{3}\left(\log_{5^{\frac{1}{3}}} x\right)} < 5^{\frac{1}{3}(6)} \; ; \quad x < 5^{\frac{1}{3}\cdot 6} \; ; \quad x < 5^{\frac{6}{3}}$$

$x < 5^2$; $\quad x < 25$: S_{lg}



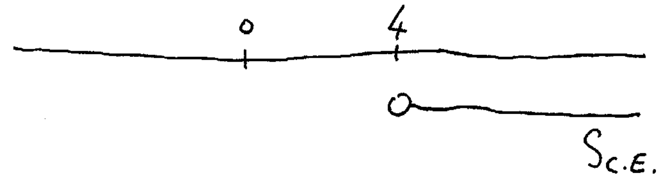


$$S_t =]0;25[$$

Esercizio N 3

$\lg_{\frac{1}{10}}(x-4) > 1$

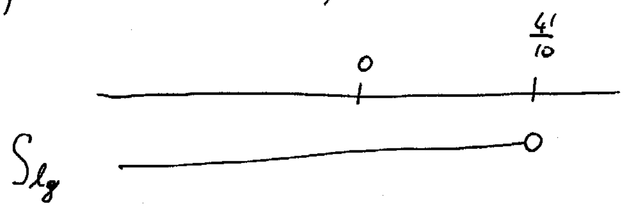
C.E. : $x-4>0$, $x>4$: $S_{C.E.}$

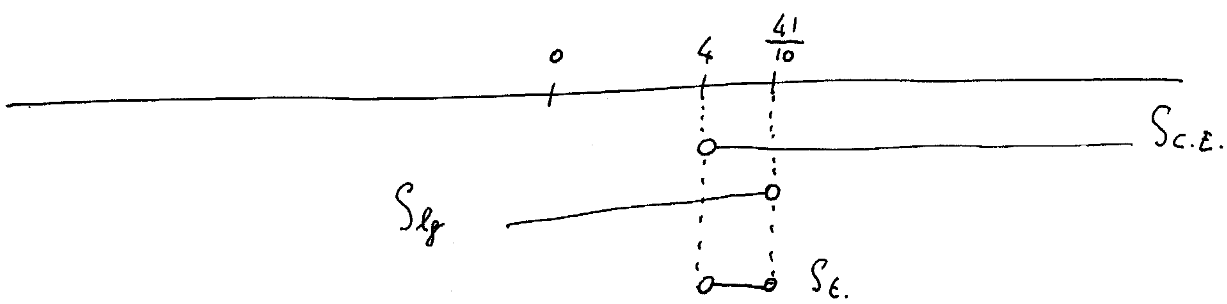


$\lg_{\frac{1}{10}}(x-4)>1$; $-\lg_{10}(x-4)>1$; $\lg_{10}(x-4)<-1$

$10^{\lg_{10}(x-4)} < 10^{-1}$; $x-4<10^{-1}$; $x-4<\frac{1}{10}$;

$x<4+\frac{1}{10}$; $x<\frac{40+1}{10}$; $x<\frac{41}{10}$. $S_{lg} : x<\frac{41}{10}$





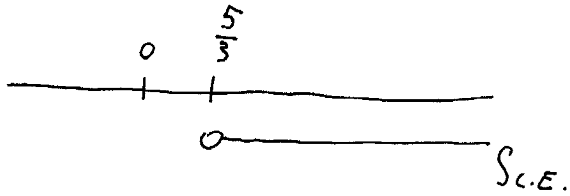
$S_t =]4 ; \frac{41}{10}[$

Esercizio N 4

P5

$$\log_{\frac{1}{2}}(3x-5) > 2$$

C.E. : $3x-5>0$, $3x>5$; $x>\frac{5}{3}$



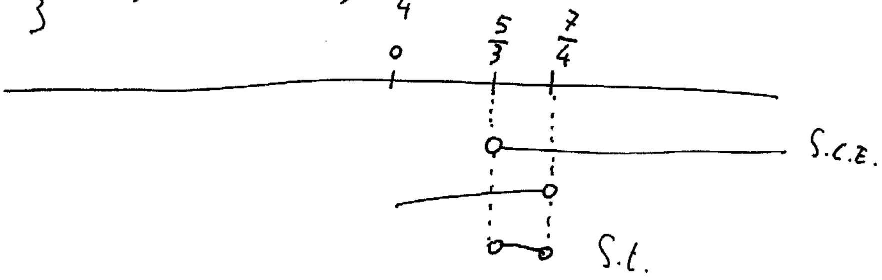
Applichiamo la funzione esponenziale di base $\frac{1}{2}$ ad ambo i membri, ricordando che essendo $\frac{1}{2}<1$, dobbiamo invertire il senso alla disequazione

$$\left(\frac{1}{2}\right)^{\log_{\frac{1}{2}}(3x-5)} < \left(\frac{1}{2}\right)^{2};\quad 3x-5 < \left(\frac{1}{2}\right)^{2};\quad 3x-5<\left(2^{-1}\right)^{2};$$

$$3x-5<2^{-2};\quad 3x<5+2^{-2};\quad 3x<5+\frac{1}{2^{2}};$$

$$3x<5+\frac{1}{4};\quad 3x<\frac{20+1}{4};\quad 3x<\frac{21}{4};\quad x<\frac{\frac{21}{4}}{3}$$

$$x<\frac{21}{4}\cdot\frac{1}{3};\quad x<\frac{\cancel{21}^{\,7}}{\cancel{12}_{\,4}};\quad x<\frac{7}{4} : S_{lg}$$



$$S_t = \left]\frac{5}{3};\frac{7}{4}\right[$$

Esercizio N5

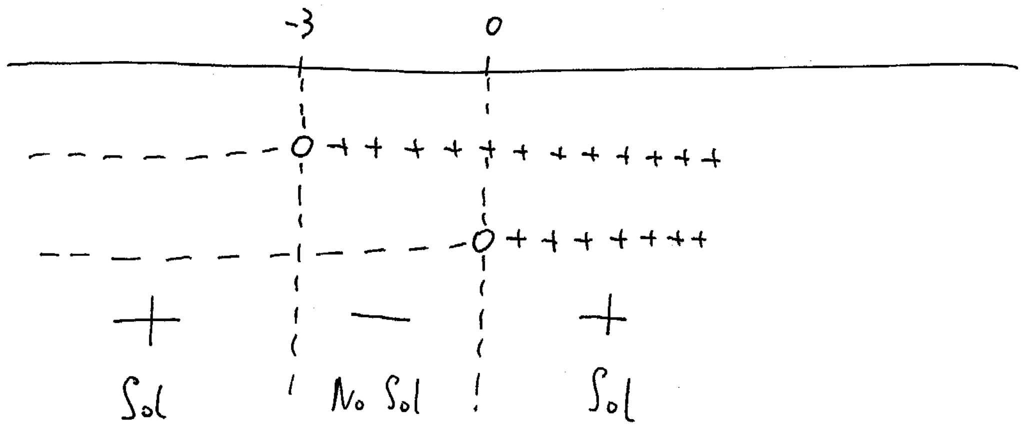
P6

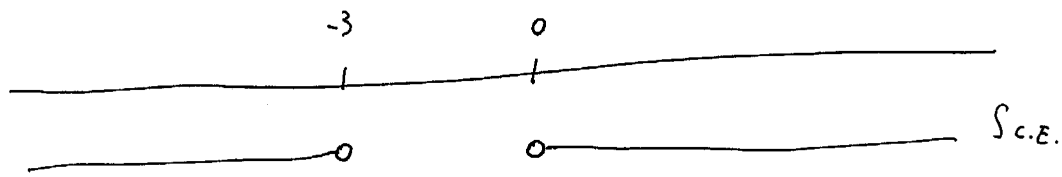
$$\log_2 \frac{x+3}{x} > 1$$

C.E. $\frac{x+3}{x} > 0$ studiamo la disequazione razionale fratta

$N(x) > 0$: $x+3>0$, $x>-3$

$D(x) > 0$: $x > 0$





Applichiamo la funzione esponenziale di base 2

$$2^{\log_2 \frac{x+3}{x}} > 2^1 \; ; \; \frac{x+3}{x} > 2 \; ; \; \frac{x+3}{x} - 2 > 0$$

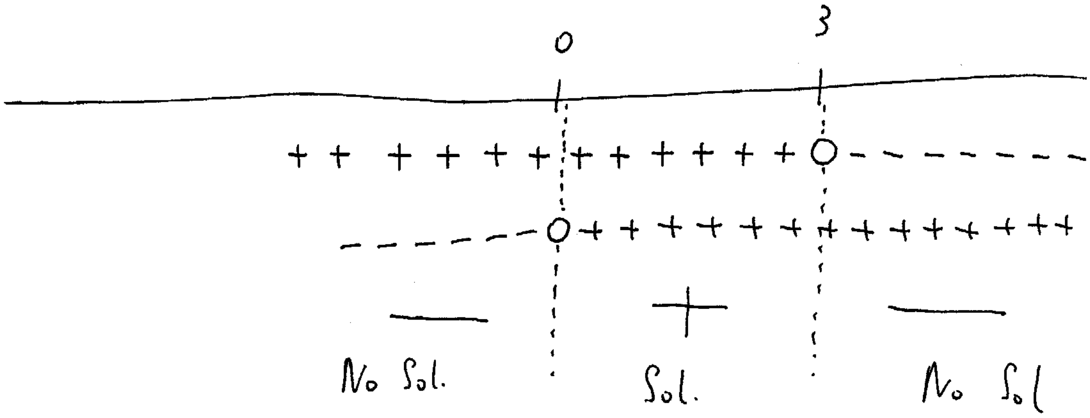
$$\frac{x+3-2x}{x} > 0 \; ; \; \frac{-x+3}{x} > 0$$

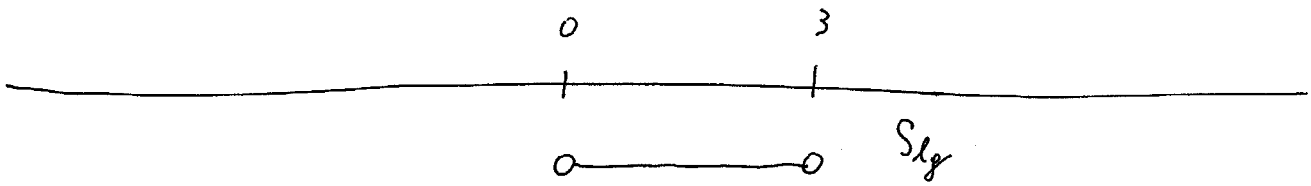
P7

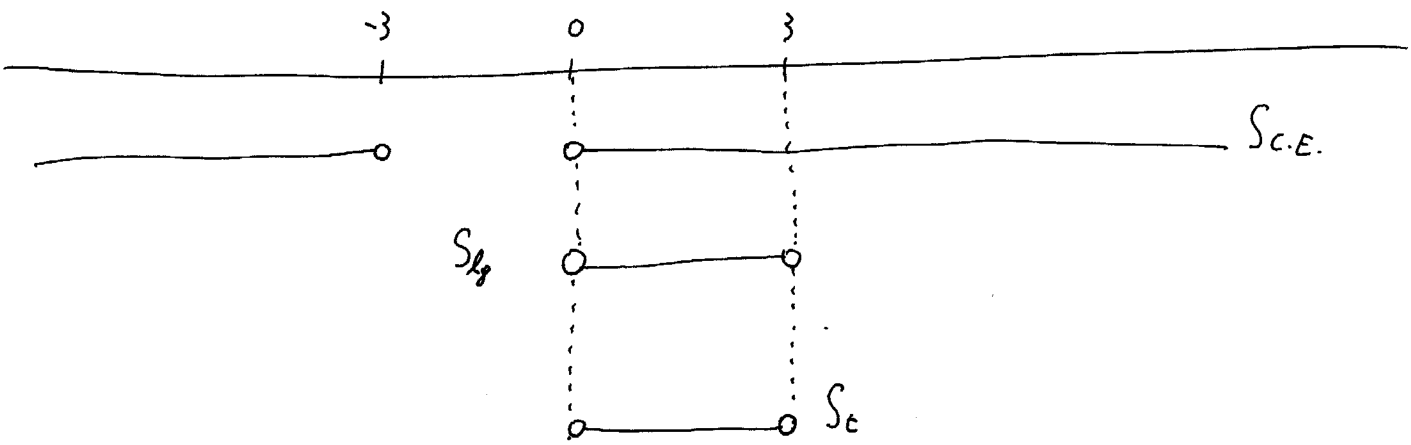
$N(x) > 0$

$-x+3>0\,;\quad 3>x\,;\quad x<3\quad$ per x minore di 3 $N(x)>0$

$D(x) > 0 \qquad x>0$







$S_t = \,]0;3[$

Esercizio N 6

$$\lg_2\left(x^2-\frac{3}{4}\right)<-2$$

C.E.: $x^2-\frac{3}{4}>0$; $\frac{4x^2-3}{4}>0$; $4x^2-3>0$

$\Delta=b^2-4ac=0-4(4)(-3)=+48$

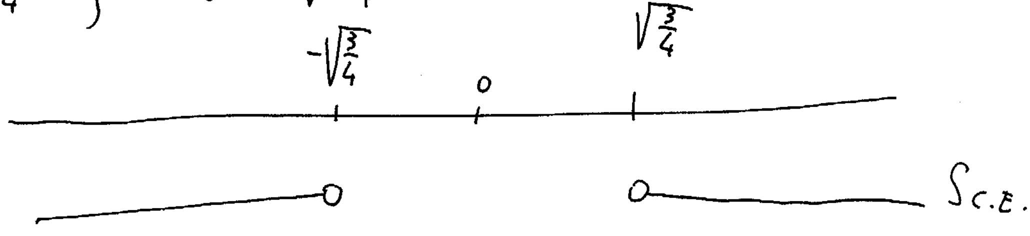
La famiglia della disequazione è >0 e il Δ è >0 $\Rightarrow$

soluzioni positive all'esterno delle radici.

Determiniamo le radici dell'equazione associata

$4x^2-3=0$; $4x^2=3$; $x^2=\frac{3}{4}$; $x=\pm\sqrt{\frac{3}{4}}$

$x_1=+\sqrt{\frac{3}{4}}$; $x_2=-\sqrt{\frac{3}{4}}$



$2^{\lg_2\left(x^2-\frac{3}{4}\right)}<2^{-2}$; $x^2-\frac{3}{4}<\frac{1}{2^2}$; $x^2-\frac{3}{4}<\frac{1}{4}$

$x^2<\frac{3}{4}+\frac{1}{4}$; $x^2<\frac{3+1}{4}$; $x^2<\frac{4}{4}$; $x^2<1$

$x^2-1<0$

$\Delta = b^2 - 4ac = 0 - 4(1)(-1) = 4$

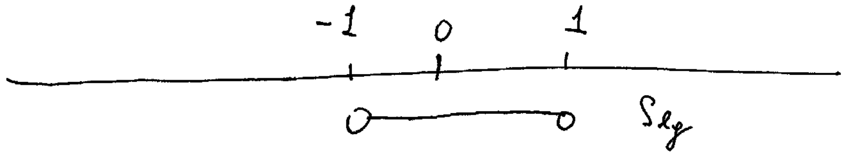
(p9)

la famiglia è del tipo < 0, il Δ è > 0 $\Rightarrow$

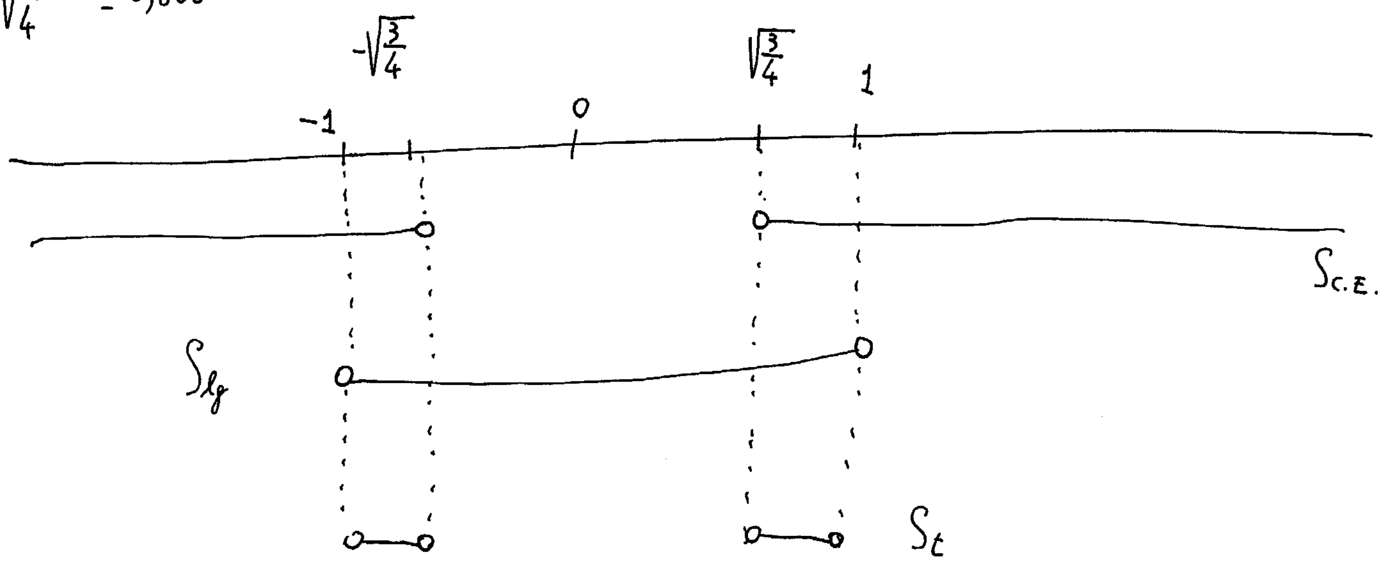
le soluzioni della disequazione sono interne

Risolviamo l'equazione associata

$x^2 - 1 = 0$; $x^2 = 1$; $x = \pm 1$



$\sqrt{\frac{3}{4}} = 0{,}866$



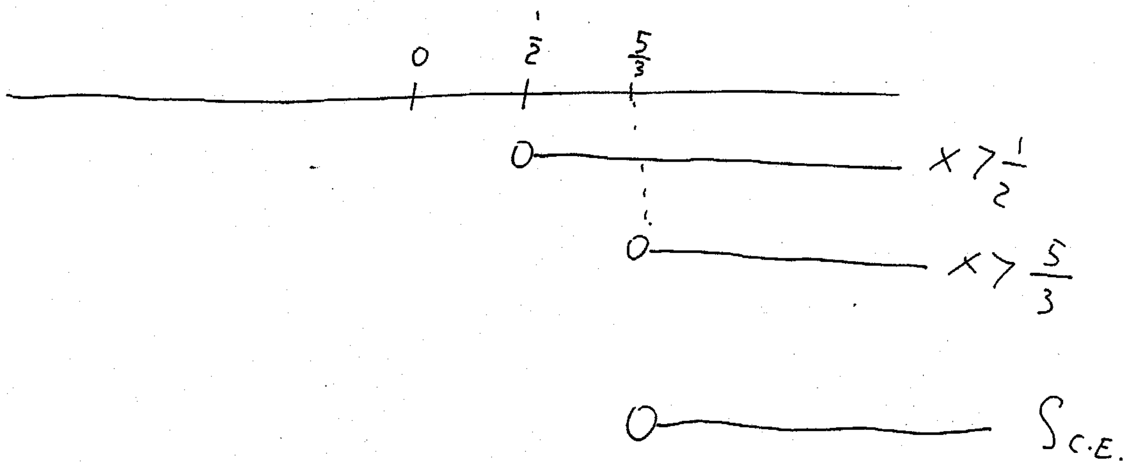
$S_t = \left]-1; -\sqrt{\frac{3}{4}}\right[\cup \left]\sqrt{\frac{3}{4}}; 1\right[$ cioè

$S_t = \left]-1; -\frac{\sqrt{3}}{2}\right[\cup \left]\frac{\sqrt{3}}{2}; 1\right[$

Esercizio N7

$$\log_{\frac{1}{2}}(3x-5) < \log_{\frac{1}{4}}(2x-1)$$

$$C.E. = \begin{cases} 3x-5>0 \\ 2x-1>0 \end{cases} \quad \begin{cases} 3x>5 \\ 2x>1 \end{cases} \quad \begin{cases} x>\frac{5}{3} \\ x>\frac{1}{2} \end{cases}$$



$$S_{C.E.} = \left]\frac{5}{3};+\infty\right[.$$

$$\log_{\frac{1}{2}}(3x-5) < \log_{\frac{1}{4}}(2x-1)$$

$$-\log_2 (3x-5) < -\log_4 (2x-1) \;;$$

$$\log_4 (2x-1) < \log_2 (3x-5)$$

Applichiamo la formula del cambiamento di base $\log_a b = \dfrac{\log_c b}{\log_c a}$

$$\cancel{\log_4 (2x-1) < \log}$$

$$\frac{\log_2 (2x-1)}{\log_2 4} < \log_2 (3x-5)$$

$$\frac{\log_2 (2x-1)}{\log_2 2^2} < \log_2 (3x-5);$$

P12

$$\frac{\log_2 (2x-1)}{2\log_2 2} < \log_2 (3x-5); \quad \frac{\log_2 (2x-1)}{2} < \log_2 (3x-5);$$

$$\log_2 (2x-1) < 2 \cdot \log_2 (3x-5); \quad \log_2 (2x-1) < \log_2 (3x-5)^2$$

Applichiamo la funzione esponenziale di base 2

$$2^{\log_2 (2x-1)} < 2^{\log_2 (3x-5)^2}; \quad 2x-1 < (3x-5)^2;$$

$$2x-1-(3x-5)^2 < 0; \quad 2x-1-(9x^2+25-30x) < 0$$

$$2x-1-9x^2-25+30x < 0; \quad -9x^2+32x-26 < 0 \quad (*)$$

moltiplichiamo $(*)$ per (-1)

$$(\square) \quad 9x^2-32x+26 > 0 \qquad \Delta = b^2-4ac = (32)^2-4(9)(26) =$$

$$= 1024 - 936 = 88$$

$$\Delta = 88$$

(0) è della famiglia > 0 e Δ è $> 0 \Rightarrow$ PB

soluzioni positive all'esterno.

Calcoliamo le soluzioni dell'equazione associata

$9x^2 - 32x + 26 = 0$

$$x = \frac{32 \pm \sqrt{88}}{18} =$$

88	2
44	2
22	2
11	11
1	

$88 = 2^3 \cdot 11$

$$= \frac{32 \pm \sqrt{2^3 \cdot 11}}{18} = \frac{32 \pm \sqrt{2^2 \cdot 2 \cdot 11}}{18} = \frac{32 \pm 2\sqrt{22}}{18} =$$

$$= \frac{\cancel{2}^{1}(16 \pm \sqrt{22})}{\cancel{18}_{9}} = \frac{16 \pm \sqrt{22}}{9}$$

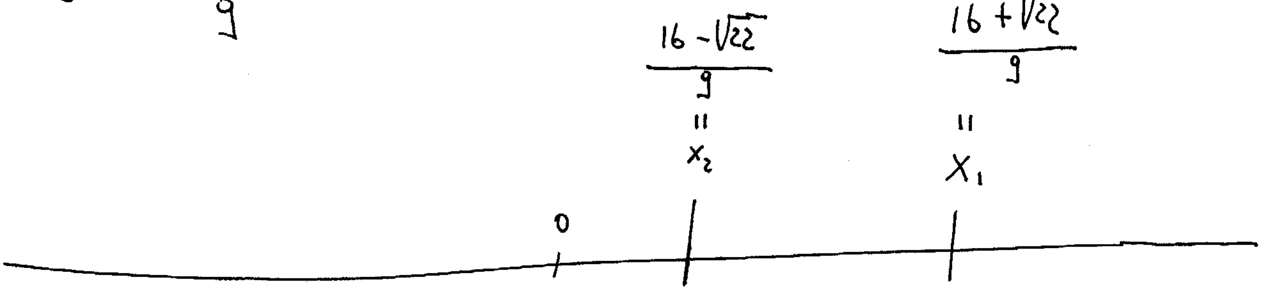
$$x_1 = \frac{16 + \sqrt{22}}{9}$$

$$x_2 = \frac{16 - \sqrt{22}}{9}$$

$$x_1 = \frac{16+4{,}7}{9} = 2{,}3$$

P 14

$$x_2 = \frac{16-4{,}7}{9} = 1{,}26$$



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Da cui (*) ha

soluzioni

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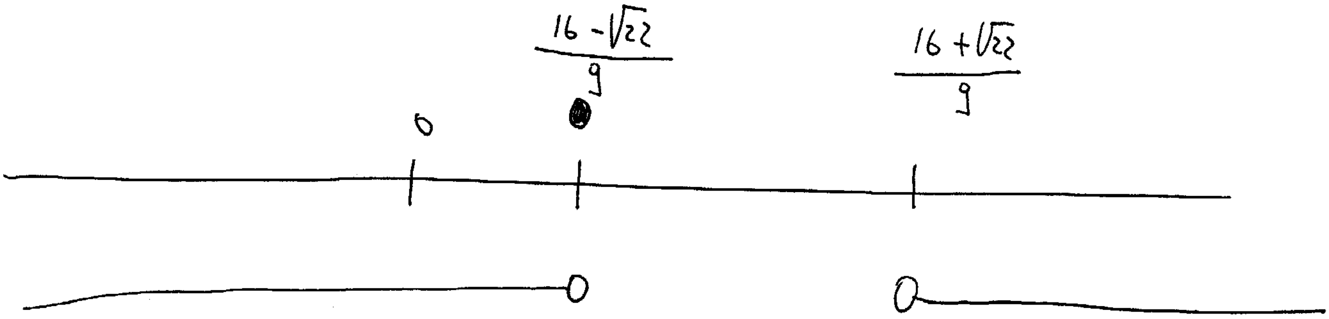
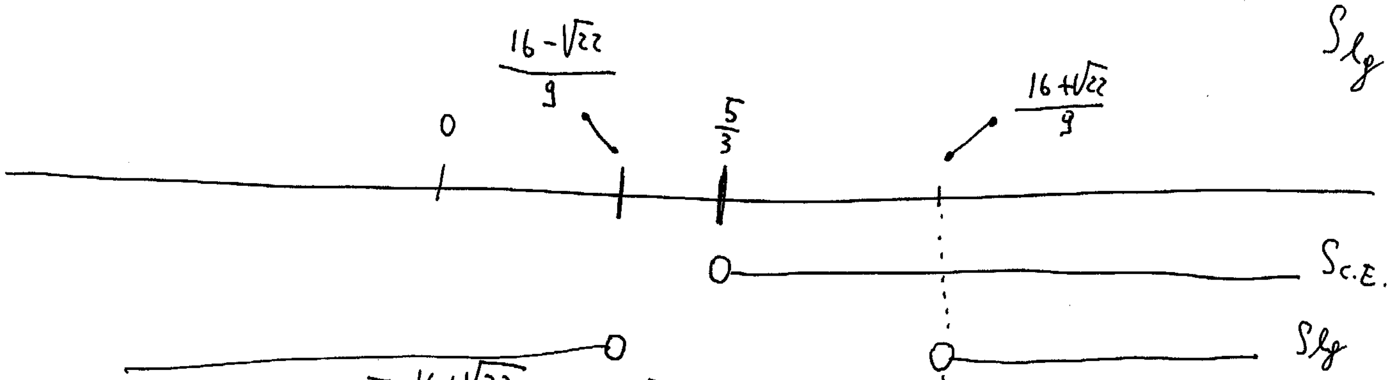
(*) cerca le soluzioni negative $\Rightarrow$



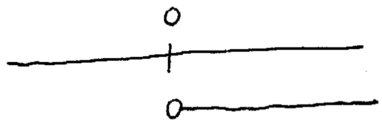


$$S_t = \left] \frac{16+\sqrt{22}}{9} ; +\infty \right[$$

Esercizio N° 8

$$3\log_2 x - \frac{12}{\log_2 x} < 5$$

C.E. $x > 0$



$S_{C.E.}$

$\log_2 x = t$

$$3t - \frac{12}{t} < 5 \; ; \; \frac{3t^2-12}{t} < \frac{5t}{t} \; ; \; \frac{3t^2-12}{t} - \frac{5t}{t} < 0 \; ;$$

$$\frac{3t^2-12-5t}{t} < 0 \; ; \; \frac{3t^2-5t-12}{t} < 0$$

$N(t) > 0$

$3t^2-5t-12 > 0 \qquad \Delta = b^2-4ac = 25-4(3)(-12) = 25+144 = 169$

$\Delta = 169$

famiglia > 0, $\Delta > 0 \Rightarrow$ soluzioni positive all'esterno

troviamo le soluzioni dell'equazione associata

$$3t^2-5t-12=0 \; ; \; t = \frac{5 \pm \sqrt{169}}{6} = \frac{5 \pm 13}{6} = \begin{cases} \frac{5+13}{6} = 3 \\ \frac{5-13}{6} = -\frac{8}{6} = -\frac{4}{3} \end{cases}$$

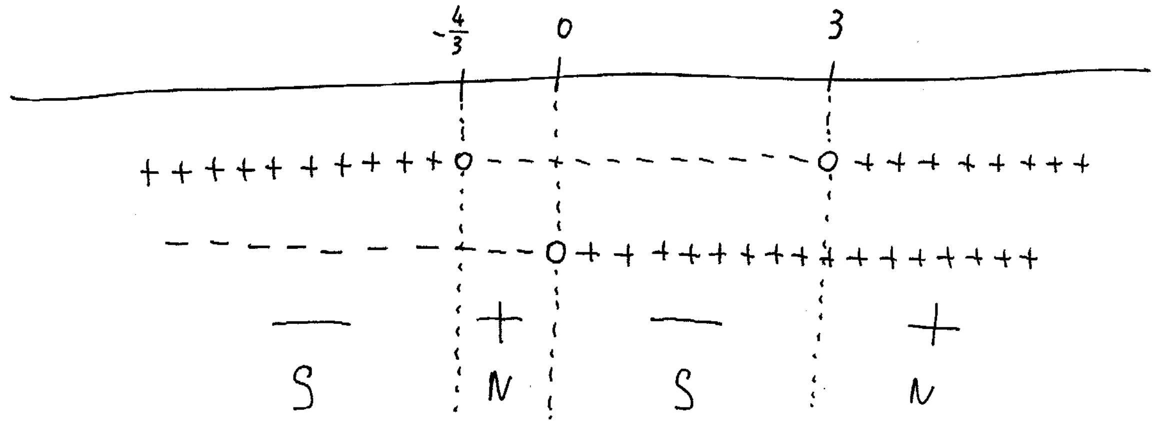
$t_1 = 3$

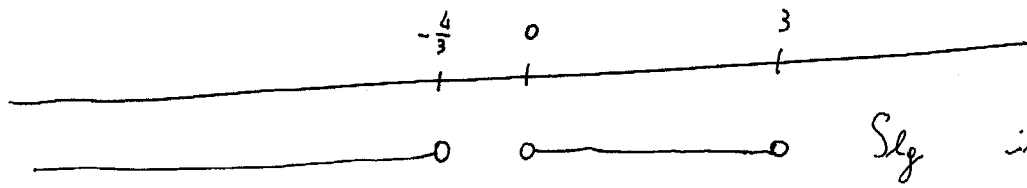
$t_2 = -\frac{4}{3}$

soluzioni positive all'esterno e negative all'interno

$D(t) > 0 \qquad t > 0$

P16





Slg in parametro t

determiniamo le soluzioni nella variabile x

sapendo che $\log_2 x = t$

Analizziamo i vari estremi degli intervalli

$-\infty, -\frac{4}{3}, 0, 3$

$\log_2 x = -\infty \implies x = 0$ •

$\log_2 x = -\frac{4}{3}$; $2^{\log_2 x} = 2^{-\frac{4}{3}}$; $x = 2^{-\frac{4}{3}}$; $x = \frac{1}{2^{\frac{4}{3}}}$;

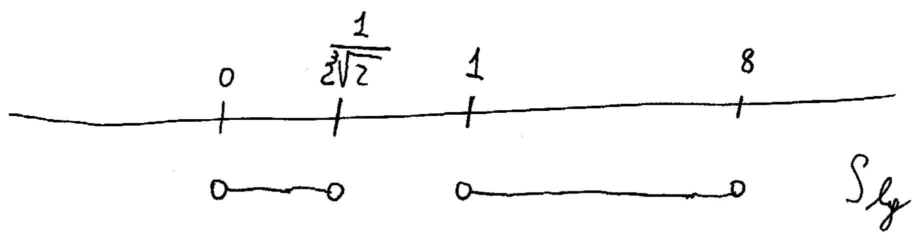
$x = \frac{1}{\sqrt[3]{2^4}}$; $x = \frac{1}{2\sqrt[3]{2}}$ •

$\log_2 x = 0$; $2^{\log_2 x} = 2^0$; $x = 2^0$; $x = 1$.

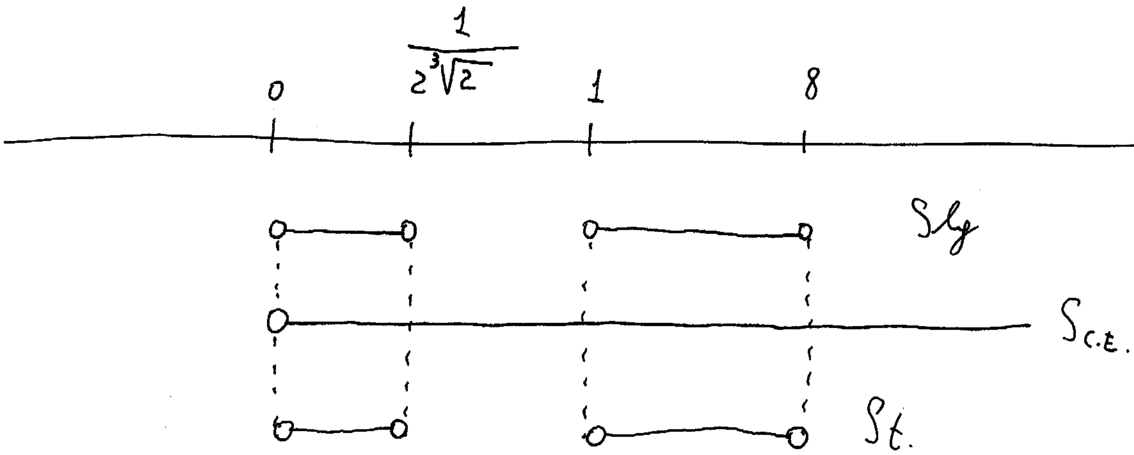
P17

$\log_2 x = 3$; $2^{\log_2 x} = 2^3$; $x = 8$.

Ricomponiamo gli intervalli



Ricerchiamo la soluzione totale



$$St = \left]0 ; \frac{1}{2\sqrt[3]{2}}\right[\cup \left]1 ; 8\right[$$