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Esercizi svolti sugli integrali

ES 1

$$\int \frac{\cos x}{\operatorname{sen}^3 x}\, dx$$

per sostituzione, poniamo $t = \operatorname{sen} x \Rightarrow$

$x = \operatorname{arcsen} t$, $t^3 = \operatorname{sen}^3 x$, $dx = \frac{1}{\sqrt{1-t^2}}\, dt$

dalla relazione fondamentale della goniometria sappiamo

che $\operatorname{sen}^2 x + \cos^2 x = 1$, $\cos^2 x = 1 - \operatorname{sen}^2 x$,

$\cos x = \sqrt{1 - \operatorname{sen}^2 x} \Rightarrow \cos x = \sqrt{1 - t^2}$

adesso abbiamo tutti gli elementi per sostituire

tutti gli elementi della ~~t~~traccia

$$\int \frac{\cancel{\sqrt{1-t^2}}}{t^3} \cdot \frac{1}{\cancel{\sqrt{1-t^2}}}\, dt =$$

②

$$= \int \frac{1}{t^3}\, dt = \int t^{-3}\, dt = \frac{1}{-3+1}\, t^{-3+1} =$$

$$= -\frac{1}{2}\, t^{-2} = -\frac{1}{2}\, \frac{1}{t^2} = -\frac{1}{2t^2} =$$

$$= -\frac{1}{2\,\mathrm{sen}^2 x} + C$$

:)

Es 2 ③

$$\int \frac{\ln^3 x}{x}\, dx$$

per sostituzione. Poniamo $t = \ln x$; $e^t = \cancel{e}^{\cancel{\ln} x}$;

$e^t = x$; per cui $dx = e^t \cdot dt$

Sostituendo si ha

$$\int \frac{t^3}{\cancel{e^t}} \cdot \cancel{e^t}\, dt = \int t^3\, dt = \frac{1}{4} t^4 =$$

$$= \frac{1}{4} \ln^4 x + C.$$

~

Es 3

$$\int \frac{\sqrt[3]{\ln x}}{3x}\, dx$$

poniamo $\ln x = t$; $e^{\ln x} = e^{t}$; $x = e^{t}$

$$dx = e^{t} \cdot dt$$

Sostituendo si ha

$$\int \frac{\sqrt[3]{t}}{3\cancel{e^{t}}} \cdot \cancel{e^{t}} \cdot dt = \int \sqrt[3]{t}\, dt =$$

$$\int t^{\frac{1}{3}}\, dt = \frac{1}{\frac{1}{3}+1} \cdot t^{\frac{1}{3}+1} =$$

$$= \frac{1}{\frac{4}{3}} \cdot t^{\frac{4}{3}} = \frac{3}{4}\sqrt[3]{t^{4}} =$$

$$= \frac{3}{4}\, t\sqrt[3]{t} = \frac{3}{4} \ln x \cdot \sqrt[3]{\ln x} + C$$

Es 4

$$\int \frac{1-\cos x}{(x-\operatorname{sen} x)^2}\,dx$$

5

poniamo $x - \operatorname{sen} x = t$; $dx - \cos x \cdot dx = dt$;

$dx(1-\cos x) = dt$; $dx = \dfrac{dt}{1-\cos x}$

Sostituiamo

$$\int \frac{dt}{t^2} = \int t^{-2}\,dt = \frac{1}{-2+1}\,t^{-2+1} =$$

$$= -t^{-1} = -\frac{1}{t} = -\frac{1}{x-\operatorname{sen} x} + C$$

~

Es 5

6

$$\int \frac{x^2}{x^3-9}\,dx$$

poniamo $x^3-9=t$, differenziamo,

$d(x^3-9)\cdot dx = dt$; $3x^2\cdot dx = dt$;

$\frac{x^2\,dx}{3} = \frac{dt}{3}$; $x^2\,dx = \frac{dt}{3}$

$$\int \frac{\frac{dt}{3}}{t} = \int \frac{1}{3}\,dt \cdot \frac{1}{t} =$$

$$= \frac{1}{3}\int \frac{1}{t}\,dt = \frac{1}{3}\ln|t| = \frac{1}{3}\ln|x^3-9| + C$$

~ Es 6

$$\int \frac{x-1}{1+x^2}\,dx =$$

$$= \int \frac{x}{1+x^2}\,dx - \int \frac{1}{1+x^2}\,dx = \int \frac{2}{2}\,\frac{x}{1+x^2}\,dx - \int \frac{1}{1+x^2}\,dx =$$

$$= \frac{1}{2}\int \frac{2x}{1+x^2}\,dx - \operatorname{arctg} x = \frac{1}{2}\ln|1+x^2| - \operatorname{arctg} x + C$$

~

$$\int \frac{\sec^2 x}{tg\, x}\, dx$$

Per definizione $\sec x = \frac{1}{\cos x} \Rightarrow \sec^2 x = \frac{1}{\cos^2 x}$

e sappiamo anche che

$$\left(tg\, x\right)' = \frac{1}{\cos^2 x}$$ da cui

$$\left(tg\, x\right)' = \sec^2 x \Rightarrow$$

$$\int \frac{\sec^2 x}{tg\, x}\, dx = \ln |tg\, x| + C$$

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$$\int \operatorname{ctg} x \, dx =$$

$$\int \frac{\cos x}{\operatorname{sen} x} \, dx \qquad \text{ma } (\operatorname{sen} x)' = \cos x$$

per cui

$$\int \frac{\cos x}{\operatorname{sen} x} \, dx = \ln |\operatorname{sen} x| + C$$

Es 9

$$\int Th(x)\,dx =$$

$$\int \frac{senh(x)}{cosh(x)}\,dx$$ ma sappiamo che $(\cosh x)' = senh\, x$

$\Rightarrow$

$$\int \frac{senh(x)}{cosh(x)}\,dx = Ln|\cosh(x)| + C$$

Es 10

$$\int x^2 \cdot \sqrt[3]{x^3-4}\; dx$$

poniamo $x^3-4=t$; $\sqrt[3]{x^3-4}=\sqrt[3]{t}$;

$x^3=t+4$; $x=\sqrt[3]{t+4}$; $x=(t+4)^{\frac{1}{3}}$;

$dx=\frac{1}{3}(t+4)^{\frac{1}{3}-1}\,dt$; $dx=\frac{1}{3}(t+4)^{-\frac{2}{3}}\,dt$

$$dx=\frac{1}{3}\,\frac{1}{(t+4)^{\frac{2}{3}}}\,dt \;;$$

$$\boxed{dx=\frac{1}{3}\,\frac{1}{\sqrt[3]{(t+4)^2}}\,dt}$$

Ricaviamo x^2 ; da $x=(t+4)^{\frac{1}{3}}$ $\Rightarrow$

$x^2=\left[(t+4)^{\frac{1}{3}}\right]^2$;

$x^2 = (t+4)^{\frac{2}{3}}$; $\boxed{x^2 = \sqrt[3]{(t+4)^2}}$

Procediamo con le sostituzioni

$$\int x^2 \cdot \sqrt[3]{x^3 - 4} \, dx =$$

$$= \int \cancel{\sqrt[3]{(t+4)^2}} \cdot \sqrt[3]{t} \cdot \frac{1}{3} \frac{1}{\cancel{\sqrt[3]{(t+4)^2}}} \, dt =$$

$$= \frac{1}{3} \int \sqrt[3]{t} \, dt = \frac{1}{3} \int t^{\frac{1}{3}} \, dt =$$

$$= \frac{1}{3} \frac{1}{\frac{1}{3} + 1} \cdot t^{\frac{1}{3} + 1} =$$

$$= \frac{1}{3} \cdot \frac{1}{\frac{4}{3}} \, t^{\frac{4}{3}} = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{4} \cdot t^{\frac{4}{3}} =$$

$$= \frac{1}{4} \, t^{\frac{4}{3}} = \frac{1}{4} \sqrt[3]{t^4} = \frac{1}{4} \, t \sqrt[3]{t} =$$

$$= \frac{1}{4} \left(x^3 - 4\right) \cdot \sqrt[3]{x^3 - 4} + C$$

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Es 11

$$\int \frac{1+\operatorname{sen}x}{(x-\cos x)^3}\,dx$$

poniamo $t = x - \cos x$

$dt = dx - (-\operatorname{sen}x)\,dx$; $dt = dx + \operatorname{sen}x\,dx$;

$$\boxed{dt = dx\,(1+\operatorname{sen}x)}$$

$$\int \frac{dt}{t^3} = \int t^{-3}\,dt = \frac{1}{-3+1}\,t^{-3+1} =$$

$$= -\frac{1}{2}\,t^{-2} = -\frac{1}{2}\,\frac{1}{t^2} = -\frac{1}{2(x-\cos x)^2} + C$$

Es 12

$$\int \frac{\operatorname{arcsen} x}{\sqrt{1-x^2}}\, dx$$

poniamo $t = \operatorname{arcsen} x$

$$dt = \frac{dx}{\sqrt{1-x^2}} \quad \Rightarrow$$

$$\int \frac{\operatorname{arcsen} x}{\sqrt{1-x^2}}\, dx = \int t \, dt = \frac{1}{2} t^2 =$$

$$= \frac{1}{2} (\operatorname{arcsen} x)^2 = \frac{1}{2} \operatorname{arcsen}^2 x + C$$

~ Es 13

$$\int \frac{2 \operatorname{arctg} x + 1}{x^2+1}\, dx$$

poniamo $\operatorname{arctg} x = t$; $x = \operatorname{tg} t$;

$$dt = \frac{1}{x^2+1}\, dx$$

per cui l'integrale diviene

$$\int \frac{2t+1}{x^2+1}\, dx = \int 2t+1\, dt = \int 2t\, dt + \int dt =$$

$$= 2\int t\, dt + t = \cancel{2} \cdot \frac{1}{\cancel{2}} t^2 + t = \operatorname{arctg}^2 x + \operatorname{arctg} x =$$

$$= \operatorname{arctg} x \left(\operatorname{arctg} x + 1\right) + C$$

~ Es 14

$$\int \cos^2 x \cdot \text{sen}^2 x \cdot \cos 2x \, dx$$

noi sappiamo che

$\text{sen}(2x) = 2 \,\text{sen}\, x \cdot \cos x$ eleviamo al quadrato

$\text{sen}^2(2x) = 4 \,\text{sen}^2 x \cdot \cos^2 x$ da cui

$$\frac{\text{sen}^2(2x)}{4} = \text{sen}^2 x \cdot \cos^2 x \quad \Rightarrow$$

$$\text{sen}^2 x \cdot \cos^2 x = \frac{1}{4} \,\text{sen}^2(2x)$$

L'integrale diviene

$$\int \cos^2 x \cdot \text{sen}^2 x \cdot \cos 2x \, dx = \int \frac{1}{4} \,\text{sen}^2(2x) \cdot \cos(2x) \, dx =$$

$$= \frac{1}{4} \int \text{sen}^2(2x) \cdot \cos(2x) \, dx =$$

$$= \frac{1}{4} \int \left[sen(2x) \right]^2 \cdot \cos(2x) \, dx = (2)$$

poniamo $sen(2x) = t$, differenziamo

$$\cos(2x) \cdot 2 \cdot dx = dt \; ; \; dx = \frac{dt}{2\cos(2x)}$$

Continuiamo ~~con~~ dalle formule (2)

$$= \frac{1}{4} \int t^2 \cdot \cos(2x) \, dx = \frac{1}{4} \int t^2 \cdot \cancel{\cos(2x)} \cdot \frac{dt}{2\cancel{\cos(2x)}} =$$

$$= \frac{1}{8} \int t^2 \, dt = \frac{1}{8} \cdot \frac{1}{3} \cdot t^3 = \frac{1}{24} t^3 =$$

$$= \frac{1}{24} sen^3(2x) + C$$

Es 15

$$\int x \cdot \ln x \, dx$$

Integriamo per parti

$$\int f(x) \cdot g'(x) \, dx = f(x) \cdot g(x) - \int f'(x) \cdot g(x) \, dx$$

$f(x)$ = fattore finito

$g'(x)$ = fattore differenziale

$f'(x)$ = differenziale del fattore finito

$g(x)$ = integrale del fattore differenziale

poniamo come fattore finito $f(x) = \ln x$

e come fattore differenziale $g'(x) = x$, per cui

$f'(x) = \frac{1}{x}$ e $g(x) = \int g'(x) \, dx = \int x \, dx = \frac{1}{2} x^2$

Applichiamo la formula dell'integrazione per parti

$$\int x \cdot \ln x \, dx = \ln x \cdot \frac{1}{2} x^2 - \int \frac{1}{\cancel{x}} \cdot \frac{1}{2} x^{\cancel{2}} \, dx =$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x \, dx = \frac{1}{2} x^2 \cdot \ln x - \frac{1}{2} \cdot \frac{1}{2} x^2 =$$

$$= \frac{1}{2} x^2 \cdot \ln x - \frac{1}{4} x^2 + C = \frac{1}{4} x^2 \left(2 \ln x - x^2 \right) + C$$

$$\int x \cdot \operatorname{sen} x \, dx$$

poniamo $f(x) = x$ e $g'(x) = \operatorname{sen} x$

$f'(x) = 1$ e $g(x) = \int g'(x)\, dx = \int \operatorname{sen} x \, dx = -\cos x$

cioè $g(x) = -\cos x$

L'integrale diviene

$$\int x \cdot \operatorname{sen} x \, dx = x \cdot (-\cos x) - \int 1 \cdot (-\cos x)\, dx =$$

$$= -x \cos x + \int \cos x \, dx = -x \cdot \cos x + \operatorname{sen} x + C$$

~ Es 17

$$\int x \cdot e^x \, dx$$

poniamo $f(x) = x$; $g'(x) = e^x$;

$f'(x) = 1$; $g(x) = e^x$

L'integrale di partenza diviene

$$\int x \cdot e^x \, dx = x \cdot e^x - \int 1 \cdot e^x \, dx =$$

$$= x e^x - e^x + C = e^x (x - 1) + C$$

ES 18

$$\int \ln x \, dx$$

poniamo

$f(x) = \ln x$, $g'(x) = 1$

Da cui

$f'(x) = \frac{1}{x}$; $g(x) = \int g'(x)\, dx = \int 1\, dx = x$

$g(x) = x$

L'integrale di partenza diviene

$$\int \ln x \, dx = x \cdot \ln x - \int \frac{1}{\cancel{x}} \cdot \cancel{x} \, dx =$$

$$= x \cdot \ln x - \int dx = x \cdot \ln x - x + C =$$

$$= x(\ln x - 1) + C$$

Es 19

$$\int x^2 \cdot \ln x \, dx =$$

$f(x) = \ln x$; $g'(x) = x^2$

Da cui $f'(x) = \frac{1}{x}$ e $g(x) = \int g'(x)\, dx =$

$= \int x^2 \, dx = \frac{1}{3} x^3$, per cui $g(x) = \frac{1}{3} x^3$.

$$\int x^2 \cdot \ln x \, dx = \ln x \cdot \frac{1}{3} x^3 - \int \frac{1}{\cancel{x}} \cdot \frac{1}{3} x^{\cancel{3}\,2} \, dx =$$

$$= \frac{1}{3} x^3 \cdot \ln x - \frac{1}{3} \int x^2 \, dx = \frac{1}{3} x^3 \cdot \ln x -$$

$$- \frac{1}{3} \cdot \frac{1}{3} \cdot x^3 = \frac{1}{3} x^3 \cdot \ln x - \frac{1}{9} x^3 + C =$$

$$= x^3 \left(\frac{1}{3} \ln x - \frac{1}{9} \right) + C$$

ES 20

$$\int x \cdot e^{2x}\, dx$$

poniamo $f(x) = x$ e $g'(x) = e^{2x}$

da cui $f'(x) = 1$ e $g(x) = \int g'(x)\, dx =$

$= \int e^{2x}\, dx$; Calcoliamo

$g(x) = \int e^{2x}\, dx$; poniamo $e^x = t$, differenziamo

$e^x\, dx = dt$ da cui $dx = \dfrac{dt}{e^x}$

torniamo all'integrale $g(x)$

$$g(x) = \int e^{2x}\, dx = \int e^{2x} \cdot \frac{dt}{e^x} =$$

$$= \int e^x \cdot \cancel{e^x}\, \frac{dt}{\cancel{e^x}} = \int e^x\, dt = \int t \cdot dt = \frac{1}{2} t^2 =$$

$$= \frac{1}{2} (e^x)^2 = \frac{1}{2} e^{2x} ;$$

Riassumiamo

$$g(x) = \frac{1}{2} e^{2x}$$

Torniamo all'integrale di partenza

$$\int x \cdot e^{2x} \, dx = x \cdot \frac{1}{2} e^{2x} - \int 1 \cdot \frac{1}{2} e^{2x} \, dx =$$

$$= x \cdot \frac{1}{2} e^{2x} - \frac{1}{2} \int e^{2x} \, d\cancel{x} = (5)$$

ma l'integrale $\int e^{2x} \, dx$ lo abbiamo già calcolato poco fa e vale

$$\int e^{2x} \, dx = \frac{1}{2} e^{2x}$$

Torniamo alla formula (5)

$$= x \cdot \frac{1}{2} e^{2x} - \frac{1}{2} \, \frac{1}{2} e^{2x} + C = \frac{1}{2} x \cdot e^{2x} -$$

$$- \frac{1}{4} e^{2x} + C = e^{2x} \cdot \left(\frac{1}{2} x - \frac{1}{4} \right) + C$$

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ES 21

$$\int \frac{1}{1+e^x}\, dx .$$

poniamo $e^x = t$, differenziamo $e^x \cdot dx = dt$,

$$\int \frac{1}{1+e^x}\, dx = \int \frac{1}{1+t} \cdot \frac{dt}{e^x} =$$

$$\int \frac{1}{1+t} \cdot \frac{dt}{t} = \int \frac{1}{t(1+t)}\, dt =$$

applichiamo la scomposizione per l'integrazione delle funzioni razionali fratte con $N(x)$ di grado 0

$$\frac{1}{t(1+t)} = \frac{A}{t} + \frac{B}{t+1} =$$

$$= \frac{A(t+1) + B \cdot t}{t(t+1)} = \frac{At + A + Bt}{t(t+1)} =$$

$$= \frac{t(A+B) + A}{t(t+1)}$$

Da cui $\begin{cases} A+B=0 \\ A=1 \end{cases}$ $\begin{cases} 1+B=0 \\ A=1 \end{cases}$ $\begin{cases} B=-1 \\ A=1 \end{cases}$

In definitiva si ha

$$\int \frac{1}{t(1+t)}\, dt = \int \frac{1}{t} + \frac{-1}{t+1}\, dt =$$

$$= \int \frac{1}{t}\, dt - \int \frac{1}{t+1}\, dt = \ln|t| - \ln|t+1| + C =$$

$$= \ln \frac{t}{t+1} = \ln \frac{e^x}{e^x+1} + C$$

ES 22

$$\int \sqrt{1-x^2}\, dx$$

poniamo $x = sen(t)$, $t = arcsen\, x$

differenziamo $dx = \cos t \cdot dt$

L'integrale diventa

$$\int \sqrt{1-x^2}\, dx = \int \sqrt{1-sen^2(t)} \cdot \cos t \cdot dt =$$

$$= \int \cos t \cdot \cos t \cdot dt = \int \cos^2 t \; dt = (1)$$

ma noi sappiamo che per le formule di duplicazione

$$\cos 2t = 2\cos^2 t - 1 \qquad \text{per cui}$$

$$2\cos^2 t = \cos 2t + 1 \; ; \; \cos^2 t = \frac{1+\cos 2t}{2}$$

da (1) si ha

$$\int \cos^2 t \cdot dt = \int \frac{1 + \cos 2t}{2}\, dt = \int \frac{1}{2} + \frac{\cos 2t}{2}\, dt =$$

$$= \int \frac{1}{2}\, dt - \int \frac{1}{2} \cos 2t\, dt = \frac{1}{2} t - \frac{1}{2} \int \cos 2t\, dt =$$

$$= \frac{1}{2} \left[t - \int \cos 2t\, dt \right] = (3)$$

Calcoliamo $\int \cos 2t\, dt$

poniamo $2t = u$, $\quad 2\,dt = du$; $\quad dt = \frac{du}{2}$

$$\int \cos 2t\, dt = \int \cos u \cdot \frac{du}{2} = \frac{1}{2} \int \cos u\, du = \frac{1}{2} \operatorname{sen} u =$$

$$= \frac{1}{2} \cdot \operatorname{sen}(2t)$$

da (3) si ha

$$= \frac{1}{2} \left[t - \frac{1}{2} \operatorname{sen}(2t) \right] = (4)$$

Ma

$\operatorname{sen} 2t = 2 \operatorname{sen} t \cos t \quad \Rightarrow$

da (4) $= \frac{1}{2}\left[t - \frac{1}{\cancel{2}} \cancel{2} \operatorname{sen} t \cos t\right] =$

$= \frac{1}{2}\left[t - \operatorname{sen} t \cos t\right] =$ (per la relazione fondamentale

della goniometria) $=$

$= \frac{1}{2}\left[t - \operatorname{sen} t \cdot \sqrt{1 - \operatorname{sen}^2 t}\right] =$

$= \frac{1}{2}\left[t - \operatorname{sen} t \cdot \sqrt{1 - \operatorname{sen} t \cdot \operatorname{sen} t}\right] =$

$=$ (essendo $t = \arcsin x$) $=$

$= \frac{1}{2}\left[\arcsin x - \operatorname{sen}(\arcsin(\cancel{x})) \cdot \sqrt{1 - \operatorname{sen}(\arcsin x) \cdot \operatorname{sen}(\arcsin x)}\right] =$

$= \frac{1}{2}\left[\arcsin x - \cancel{x} \cdot \sqrt{1 - x \cdot x}\right] =$

$= \frac{1}{2}\left[\arcsin x - x\sqrt{1 - x^2}\right] + C$:)

oppure

$$= \frac{1}{2} \operatorname{arcsen} x - \frac{1}{2} x \sqrt{1 - x^2} + C$$

ES 23

$$\int \frac{x-8}{\sqrt[3]{x}-2}\,dx$$

Dobbiamo trovare il fattore razionalizzante conveniente

$$\sqrt[3]{x} - \sqrt[3]{2^3} \cdot \frac{\sqrt[3]{x^2}+\sqrt[3]{8x}+\sqrt[3]{64}}{\sqrt[3]{x^2}+\sqrt[3]{8x}+\sqrt[3]{64}}$$

per cui si ha

$$\int \frac{x-8}{\sqrt[3]{x}-2}\,dx = \int \frac{x-8}{\sqrt[3]{x}-2} \cdot$$

$$\cdot \frac{\sqrt[3]{x^2}+\sqrt[3]{8x}+\sqrt[3]{64}}{\sqrt[3]{x^2}+\sqrt[3]{8x}+\sqrt[3]{64}}\,dx$$

(34)

$\sqrt[3]{x} = t$; $x^{\frac{1}{3}} = t$;

$\frac{1}{3} x^{\frac{1}{3}-1} dx = dt$; $\frac{1}{3} x^{\frac{1-3}{3}} dx = dt$;

$\frac{1}{3} x^{-\frac{2}{3}} dx = dt$; $\frac{1}{3 x^{\frac{2}{3}}} dx = dt$

$dx = 3 \cdot x^{\frac{2}{3}} \cdot dt$

ma se $x^{\frac{1}{3}} = t \Rightarrow x^{\frac{2}{3}} = t^2 \Rightarrow$

$dx = 3t^2 \cdot dt$. Se $x^{\frac{1}{3}} = t \Rightarrow x = t^3$

Torniamo all'integrale

$$\int \frac{x-8}{\sqrt[3]{x} - 2} dx = \int \frac{t^3 - 8}{t - 2} \cdot 3t^2 \, dt$$

$$
\begin{array}{rrrr|l}
t^3 & 0t^2 & 0t^1 & -8 & t-2 \\
\cline{5-5}
-t^3 & +2t^2 & & & t^2+2t+4 \\
\hline
/ & 2t^2 & 0t & -8 & \\
 & -2t^2 & +4t & & \\
\cline{2-4}
 & / & 4t & -8 & \\
 & & -4t & +8 & \\
\cline{3-4}
 & & / & / &
\end{array}
$$

$$(t^3-8) = (t-2)(t^2+2t+4)$$

$$\frac{t^3-8}{t-2} = t^2+2t+4$$

Torniamo all'integrale

$$\int (t^2+2t+4)\cdot(3t^2)\,dt =$$

$$= \int 3t^4+6t^3+12t^2\,dt = 3\int t^4\,dt + 6\int t^3\,dt + 12\int t^2\cdot dt =$$

$$= 3\cdot\frac{1}{5}t^5 + \overset{3}{\cancel{6}}\,\frac{1}{\underset{2}{\cancel{4}}}\,t^4 + \overset{4}{\cancel{12}}\cdot\frac{1}{\cancel{3}}\,t^3 =$$

$$= \frac{3}{5}t^5 + \frac{3}{2}t^4 + 4t^3 = \frac{3}{5}\left(\sqrt[3]{x}\right)^5 + \frac{3}{2}\left(\sqrt[3]{x}\right)^4 +$$

$$+4\left(\sqrt[3]{x}\right)^3 = \frac{3}{5}\sqrt[3]{x^5} + \frac{3}{2}\sqrt[3]{x^4} + 4\sqrt[3]{x^3} =$$

$$= \frac{3}{5}\,x\sqrt[3]{x^2} + \frac{3}{2}\,x\sqrt[3]{x} + 4x \Rightarrow$$

$$\int \frac{x-8}{\sqrt[3]{x}-2}\,dx = \frac{3}{5}\,x\sqrt[3]{x^2} + \frac{3}{2}\,x\sqrt[3]{x} + 4x + C$$

ES 24

$$\int \sqrt{\operatorname{sen} x} \cdot \cos^3 x \, dx$$

procediamo per sostituzione $\operatorname{sen} x = t$

differenziamo $\cos x \cdot dx = dt$

$$\int \sqrt{\operatorname{sen} x} \cdot \cos^2 x \cdot \cos x \, dx =$$

$$= \int \sqrt{t} \cdot \cos^2 x \cdot dt =$$

$$= \int \sqrt{t} \cdot (1 - \operatorname{sen}^2 x) \, dt =$$

$$\int \sqrt{t} \cdot (1 - t^2) \, dt = \int \sqrt{t} - \sqrt{t} \cdot t^2 \, dt =$$

$$= \int t^{\frac{1}{2}} - t^{\frac{1}{2}} \cdot t^{2} \, dt =$$

$$= \int t^{\frac{1}{2}} \, dt - \int t^{\frac{1+4}{2}} \, dt = \int t^{\frac{1}{2}} \, dt - \int t^{\frac{5}{2}} \, dt =$$

$$= \frac{1}{\frac{1}{2}+1} \, t^{\frac{1}{2}+1} - \frac{1}{\frac{5}{2}+1} \, t^{\frac{5}{2}+1} =$$

$$= \frac{1}{\frac{1+2}{2}} \, t^{\frac{1+2}{2}} - \frac{1}{\frac{5+2}{2}} \cdot t^{\frac{5+2}{2}} =$$

$$= \frac{1}{\frac{3}{2}} \, t^{\frac{3}{2}} - \frac{1}{\frac{7}{2}} \cdot t^{\frac{7}{2}} =$$

$$= \frac{2}{3} \, t^{\frac{3}{2}} - \frac{2}{7} \, t^{\frac{7}{2}} = \frac{2}{3} \sqrt[2]{t^{3}} - \frac{2}{7} \sqrt[2]{t^{7}} =$$

$$= \frac{2}{3} \, t \cdot \sqrt[2]{t} - \frac{2}{7} \, t^{3} \sqrt[2]{t} =$$

$$= \frac{2}{3} \operatorname{sen} x \cdot \sqrt{\operatorname{sen} x} - \frac{2}{7} \cdot \operatorname{sen}^3 x \sqrt{\operatorname{sen} x} + C =$$

$$= \operatorname{sen} x \cdot \sqrt{\operatorname{sen} x} \cdot \left(\frac{2}{3} - \frac{2}{7} \cdot \operatorname{sen}^2 x \right) + C$$

$$\int \frac{dx}{\sqrt{(1+x^2)^3}}$$

poniamo $x = tg(t)$; $t = arctg(x)$;

$$dx = \frac{1}{\cos^2 t}\, dt$$

$1+x^2$ per la sostituzione diventa

$$1+x^2 = 1+tg^2(t) = 1+\frac{sen^2 t}{\cos^2 t} = \frac{\cos^2 t + sen^2 t}{\cos^2 t} = \frac{1}{\cos^2 t}$$

per cui

$$\sqrt{\left(\frac{1}{\cos^2 t}\right)^3} = \sqrt{\left(\frac{1}{\cos^2 t}\right)^2 \cdot \left(\frac{1}{\cos^2 t}\right)} =$$

$$= \left(\frac{1}{\cos^2 t}\right) \cdot \sqrt{\frac{1}{\cos^2 t}} = \frac{1}{\cos^2 t} \cdot \frac{1}{\cos t}$$

per cui si ha

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$$\int \frac{1}{\frac{1}{\cos^2 t} \cdot \frac{1}{\cos t}} \quad \frac{1}{\cos^2 t} \, dt =$$

$$= \int \frac{1}{\frac{1}{\cos^3 t}} \cdot \frac{1}{\cos^2 t} \, dt =$$

$$= \int \cos^{\cancel{3}} t \cdot \frac{1}{\cancel{\cos^2 t}} \, dt = \int \cos t \, dt = \operatorname{sen} t =$$

$$= \operatorname{sen}\left(\operatorname{arctg} x\right) + C$$